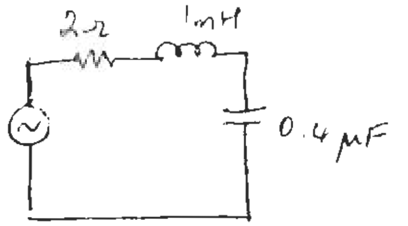


Find resonant frequency
half-power frequencies
quality factor
bandwidth
Amplitude of the current
at ω_0, ω_{c1} and ω_{c2}

The resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}} = 50 \text{ k rad/s}$$

The lower half-power frequency is

$$\omega_{c1} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$= -\frac{2}{2 \times 10^{-3}} + \sqrt{(10^3)^2 + (50 \times 10^3)^2}$$

$$= -1 + \sqrt{1 + 2500} = 49 \text{ k rad/s}$$

upper half-power frequency is

$$\omega_{c2} = 1 + \sqrt{1 + 2500} \text{ krad/s} = 51 \text{ krad/s}$$

Bandwidth $\beta = \omega_{c2} - \omega_{c1} = 2$ krad/s

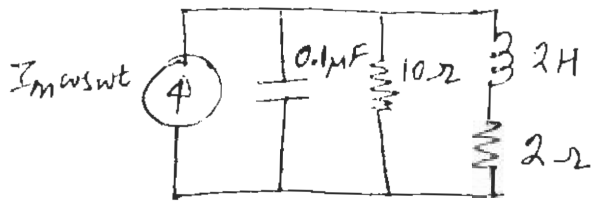
or $\beta = \frac{R}{L} = \frac{2}{10^{-3}} = 2$ krad/s

Q factor is $Q = \frac{\omega_0}{\beta} = \frac{50}{2} = \underline{\underline{25}}$.

At $\omega = \omega_0$, $I = \frac{V_m}{R} = \frac{20}{2} = 10$ A.

At $\omega = \omega_{c1}, \omega_{c2}$; $I = \frac{V_m}{\sqrt{2}R} = 7.071$ A

Determine the resonant frequency of the following circuit :



Shorter way
The input admittance is

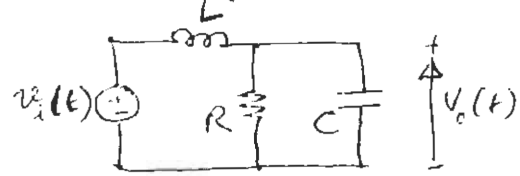
$$Y = j\omega 0.1 + \frac{1}{10} + \frac{1}{2+j\omega 2} = 0.1 + j\omega 0.1 + \frac{2-j\omega 2}{4+4\omega^2}$$

At resonance, the imaginary part of Y is 0.

$$\therefore \quad \omega_0 0.1 - \frac{2\omega_0}{4+4\omega_0^2} = 0$$

$$\Rightarrow \quad \omega_0 = 2 \text{ rad/s}.$$

Determine what type of filter is shown below:



Then calculate the corner or cut-off frequency.

Take $R = 2\,k\Omega$, $L = 2\,H$, $C = 2\,\mu F$

Solution:

We need to find the transfer function

$$H(s) = \frac{V_o}{V_i} = \frac{R \parallel 1/sC}{sL + R \parallel 1/sC}$$

$$= \frac{R/(1+sRC)}{sL + R/(1+sRC}$$

$$= \frac{R}{s^2RLC + sL + R}$$

But $s = j\omega$, $\therefore$ $H = \dfrac{R}{\sqrt{(R - \omega^2 RLC)^2 + \omega^2 L^2}}$

From $H(s)$ it is clear that it is a Second-order LOW PASS filter.

The corner frequency (half-power frequency) is where $H(s)$ is reduced to $1/\sqrt{2}$

$$\therefore H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

or $2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$

Substituting values of R, L, and C, we get

$$2 = (1 - \omega_c^2 4 \times 10^{-6})^2 + (\omega_c 10^{-3})^2$$

Assuming ω_c is in krad/s

$$2 = (1 - 4\omega_c^2)^2 + \omega_c^2$$

or $$16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

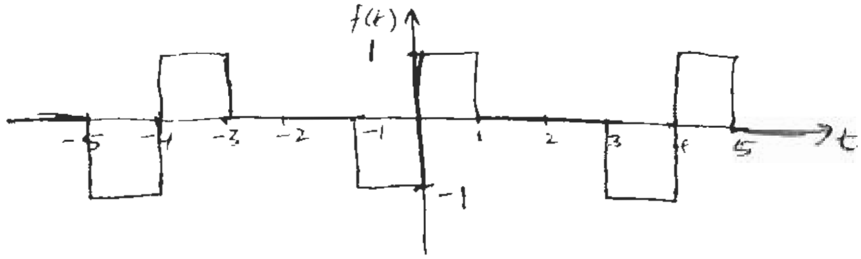
$$\Rightarrow \omega_c^2 = 0.5509 \quad \text{or} \quad -0.1134$$

$\therefore$ since ω_c^2 can only be positive,

$$\omega_c = \sqrt{0.5509} = 0.742 \text{ krad/s} = 742 \text{ rad/s}.$$

~~Design~~ ~~lowpass~~

Find the Fourier series expansion of $f(t)$ for the waveform shown below.



Solution :

The function $f(t)$ is an odd function.

$\therefore \quad a_0 = 0 = a_n$.

The period is $T = 4$

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t).\sin n\omega_0 t.\,dt$$

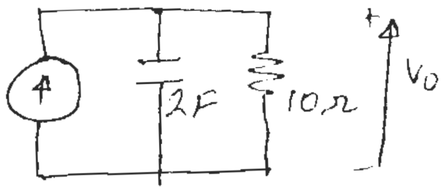
$$= \frac{4}{4}\left[\int_0^1 1.\sin\frac{n\pi}{2}t\,dt + \int_1^2 0.\sin\frac{n\pi}{2}.dt\right]$$

$$= \frac{-2}{n\pi}\cos\frac{n\pi t}{2}\Big|_0^1 = \frac{2}{n\pi}\left(1-\cos\frac{n\pi}{2}\right)$$

Hence,

$$f(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(1-\cos\frac{n\pi}{2}\right)\sin\frac{n\pi}{2}.t$$

Determine the average power supplied to the circuit in Fig. below if $i(t) = 2 + 10\cos(t + 10^\circ) + 6\cos(3t + 35^\circ)$ A



Solution: Input Impedance $= 10 \parallel \frac{1}{j2\omega} = \frac{10}{1 + j20\omega}$.

$$V = IZ = \frac{10 \cdot I}{\sqrt{1 + 400\omega^2}\,\angle \tan^{-1} 20\omega}$$

For dc component, $\omega = 0$

$I = 2A \Rightarrow V = 20V$.

For $\omega = 1$ rad/s

$$I = 10\angle 10^\circ \Rightarrow V = \frac{10(10\angle 10^\circ)}{\sqrt{1 + 400}\,\angle \tan^{-1} 20}$$

$$= 5\angle -77.14^\circ$$

For $\omega = 3$ rad/s

$$I = 6\angle 35^\circ \Rightarrow V = \frac{10(6\angle 35^\circ)}{\sqrt{1 + 3600}\,\angle \tan^{-1} 60}$$

$$= 1\angle -54.04^\circ$$

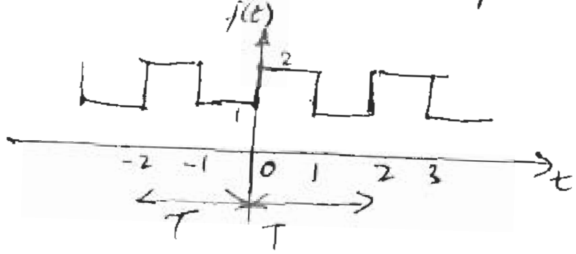
Thus, $v(t) = 20 + 5\cos(t - 77.14^\circ) + 1\cos(3t - 54.04^\circ)$ V

$$P = V_{dc} I_{dc} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

$$= 20(2) + \frac{1}{2}(5)(10) \cdot \cos[77.14^\circ - (-10^\circ)] + \frac{1}{2}(1)(6) \cdot \cos[54.04^\circ - (-35^\circ)]$$

$$= 40 + 1.247 + 0.05 = \underline{\underline{41.5\,W}}$$

Find the Fourier Series expansion of f(t) for the waveform



(Half-wave Symmetry)

The average value of the waveform = $\frac{2+1}{2} = \frac{3}{2}$

$$b_n = \frac{4}{T_0}\int_0^{T/2} \frac{1}{2} \sin n\omega_0 t \, dt = \frac{2}{T}\left(\frac{-1}{n\omega_0} \cos n\omega_0 t \Big|_0^{T/2} \right)$$

$$= \frac{-2}{n\omega_0 T_0} (\cos n\pi - 1)$$

$$= \frac{2}{n\pi} \quad \text{for } n = \text{odd}$$

$$\therefore \quad f(t) = \frac{3}{2} + \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{2}{n\pi} \sin n\omega_0 t .$$